“Bose-Einstein Condensate”

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Motivation

- A deep and application of study of statistical/quantum mechanics.
- To learn more about experimental techniques for trapping and cooling.
- To observe the stages in which matter changes its behavior.
- To increase my knowledge of physics that is present around us.

Introduction

- Bose-Einstein distribution.
- Density States.
- Condensation of a harmonically ideal gas .
- Spatial and momentum densities distributions.
- Real BEC.
- Road away to BEC.

Bose-Einstein distribution

- Bose’s publication - “*Plancks Gesetz und Lichtquantenhypothese*”.

- Einstein’s efforts to describe quantization of ideal gas.

Bose-Einstein distribution

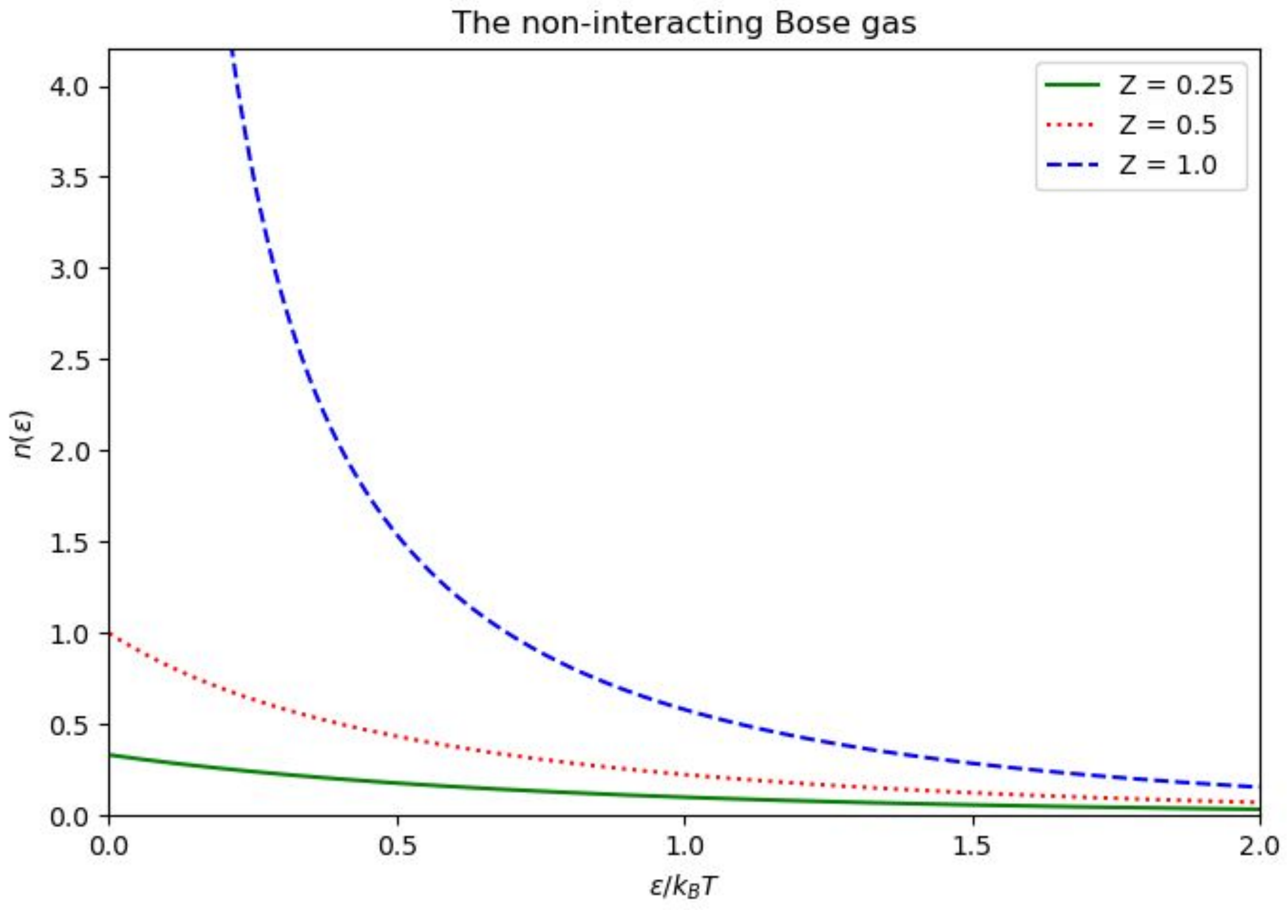
Bose-Einstein distribution →

$$n_j = \frac{g_j}{e^{\beta(\varepsilon_j - \mu)} - 1}$$

Fugacity →

$$Z = e^{\beta\mu}$$

Bose-Einstein distribution



Density of states

- A way to describe how is the distribution of energy of a system.
- Sums by integrals ⟶ is neglected N of the lowest level.
- Trapping potentials causes non-homogeneous behaviors.

Density of states

$$\int \eta(\epsilon)d\epsilon = \frac{1}{(2\pi)^3}\int d^3k d^3r.$$
$$= \frac{(2m)^{3/2}}{(2\pi)^2\hbar^3}\int d^3r \int d\epsilon\sqrt{\epsilon - U(\mathbf{r})}.$$

→ Arbitrary potential.

$$\eta(\epsilon) = \frac{\epsilon^2}{2(\hbar\bar{\omega})^3}.$$

→ Harmonic potential.

BEC - Condensation of a harmonically ideal gas

- Consequence of Bose-Einstein distribution.
- Macroscopical quantum effects.
- Temperature ⟷ Kinetic energy.
- He superfluid.

Bose-Einstein Condensate

- Number of particles in GCE → $$n_j = -\frac{\partial \Omega}{\partial \mu} = w_{T,\mu}(\varepsilon_j).$$

- When temperature decreases: $$T \to 0$$

- The system tends to occupy the lowest energy level:

$$n_0 \overset{\varepsilon_0 \to 0}{\to} w_{T,\mu}(0)$$

Bose-Einstein Condensate

- The number of particles can be given by:

$$N = \frac{1}{1/Z - 1}$$

- No thermodynamic limit: *Z = 1*. This factor express a macroscopic condition for population of ground state.

Bose-Einstein Condensate

For a specific gas in a box potential. → $$\eta(\epsilon) = \frac{(2m)^{3/2}}{(2\pi)^2\hbar^3} V\sqrt{\epsilon}.$$

The population in GS. → $$N = \int_0^\infty \eta(\epsilon) w_{T,\mu}(\epsilon) d\epsilon = \frac{V}{\lambda_{\text{th}}^3} g_{3/2}(Z).$$

Bose-Einstein Condensate

Bose-Einstein dist. is limited by phase space: $g_{3/2}(0) \rightarrow g_{3/2}(1).$

When: $T \rightarrow 0$

$$Z \overset{T\rightarrow 0}{\rightarrow} 1 \quad \text{and} \quad N \rightarrow 0$$

Bose-Einstein Condensate

- Previously cited consequence of replacing of sums by integrals.

- Rearrange the population, considering this fact:

$$N = N_c + \frac{V}{\lambda_{\text{th}}^3} g_{3/2}(Z).$$

Critical temperature - T_C

- This temperature plays a important role to describe the normal and condensation fractions.

- When: $T > T_c \quad \rightarrow \quad N_c = 0.$

- When: $T \leq T_c \quad \rightarrow \quad N = \frac{V}{\lambda_c^3} g_{3/2}(1).$

Critical temperature - T_C

- We can define $\boldsymbol{T_c}$ in terms of λ_C: $\lambda_c \equiv \sqrt{\frac{2\pi\hbar^2}{mk_BT_c}}.$
- We found:

$$T_c = \frac{2\pi\hbar^2}{k_B m}\left(\frac{N}{V g_{3/2}(1)}\right)^{2/3}.$$

Critical temperature - T_C

- Number of condensate atoms normalized by total number is given by:

$$\frac{N_c}{N} = 1 - \left(\frac{\min(T, T_c^{(3/2)})}{T_c^{(3/2)}} \right)^{3/2}.$$

Density and momentum distribution for a Bose gas

Spatial density → $$n(\mathbf{x}) = \frac{1}{\lambda_{\mathrm{th}}^3} g_{3/2}\left(e^{-\beta[U(\mathbf{x})-\mu]}\right),$$

Momentum density → $$n(\mathbf{k}) = \frac{a_{\mathrm{ho}}^6}{\lambda_{\mathrm{th}}^3} g_{3/2}\left(e^{\beta\left(\mu-\frac{\mathbf{p}^2}{2m}\right)}\right).$$

Density and momentum distribution for a Bose gas

- 10^4- 10^7 atoms at μK scale.
- Absorption Imaging and Phase-Contrast Imaging.
- BEC presents high densities, then A.I. is not a good choice.

Real BEC

- $n\lambda^3_{th} \geq 2.612$.
- Collisions are the reason to form BEC.
- The new formalism is necessary to describe interactions.
- Scattering theory and Gross-Pitaevskii equation.

Real BEC

- Solving the Schrodinger equation using *partial analysis:*

$$\sigma = \frac{4\pi}{k^2} \sum_{l}^{\infty} (2l+1) \sin^2 \delta_l .$$

- In the GS, elastic collisions plays the most important role - ***scattering length***:

$$a = - \lim_{k \to 0} \frac{\delta_0}{k} .$$

Real BEC

- Gross-Pitaevskii equation is given by:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V_t(\mathbf{R}) + NV_{\text{int}}|\Psi(\mathbf{R})|^2\right]\Psi(\mathbf{R}) = E_N\Psi(\mathbf{R})$$

- Being the interaction potential written as:

$$V_{\text{int}} = \frac{4\pi\hbar^2 a}{M}$$

Role of scattering length in BEC

- When: $a > 0.$ → BEC tends to disperse.
- When: $a < 0.$ → BEC tends to form.

Road away to BEC

- He satisfies Gross-Pitaevskii equation, but does not present a considerable account of condensate atoms.

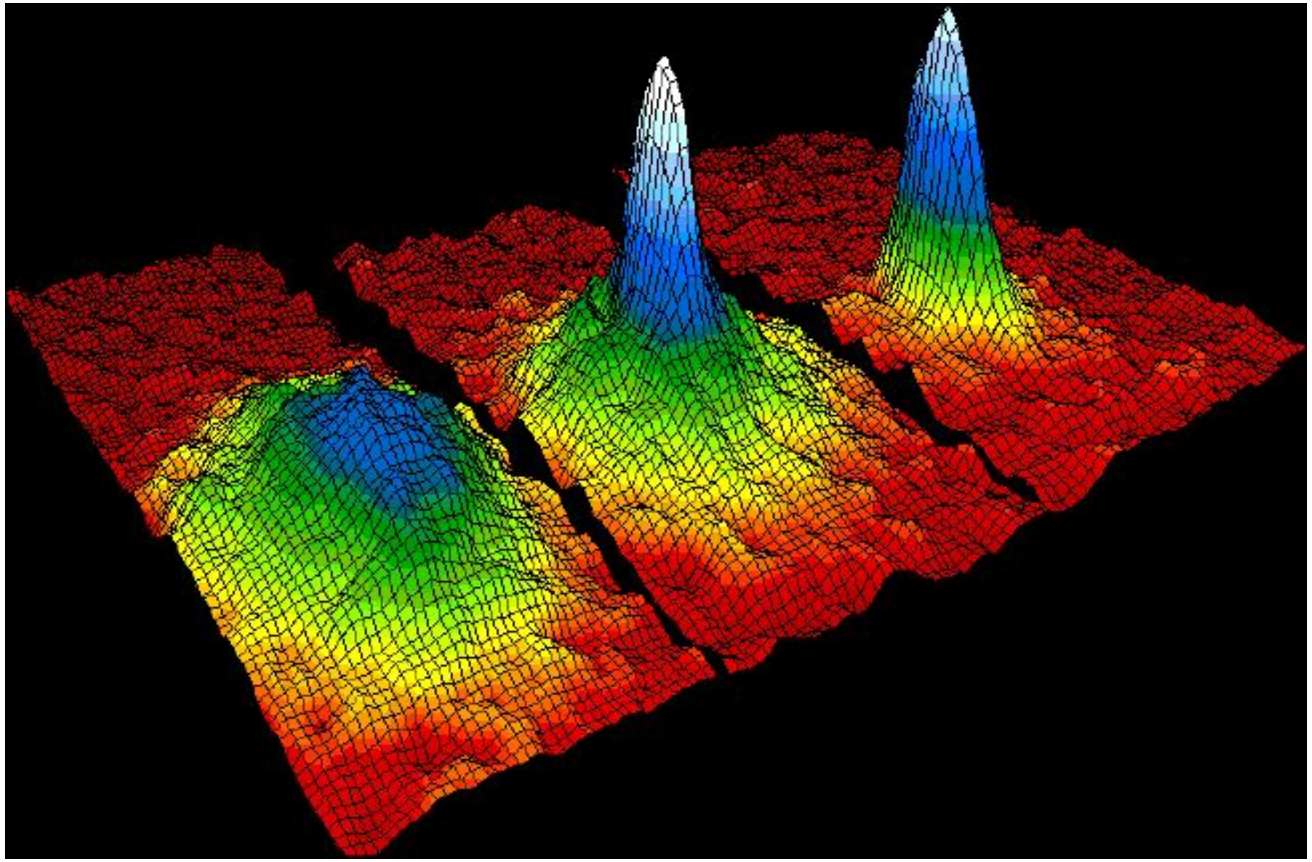
- Evaporative cooling is a fundamental technique .

- Differents types of experimental techniques: Optical Molasses, MOT, Evaporative Cooling, A.I., PCI., etc.

Road away to BEC

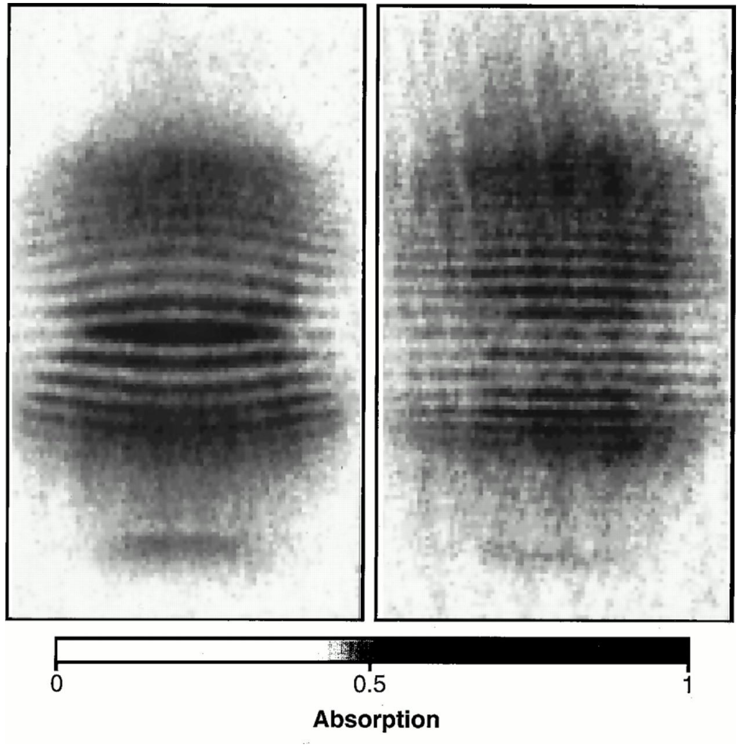
- Rubidium, sodium, lithium, and helium were the first atoms to exhibit a Bose-Einstein condensate (BEC) between 1995 - 1998.
- Cornell, Wieman for ^{88}Rb,and Keterlle for ^{23}Na, were awarded the 2001 Nobel Prize.
- Different studies have demonstrated the influence of BECs, such as the formation of quantized vortices.

Road away to BEC



Spatial distributions found from Cornell and Wieman after release from trap.
SOURCE: JILA website.

Road away to BEC



Mater-Waves interference demonstrate by Ketterle .
SOURCE: Science - 275, 637641 (1997)

Obrigado!

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